CAN MEAN VALUES OF HELMERT'S GRAVITY ANOMALIES BE CONTINUED DOWNWARD DIRECTLY?

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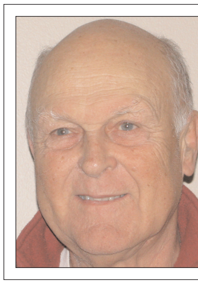
The computation of a precise gravimetric geoid based on the Stokes-Helmert approach requires the solution of the geodetic boundary value problem. For that, the mean Helmert's gravity anomaly on the earth's topographic surface must be reduced to the geoid, the surface that plays the role of the boundary. This reduction is a process known as downward continuation. This paper considers the downward continuation as a solution of the discrete inverse Poisson problem. It shows the derivation of a doubly-averaged upward continuation operator that relates mean Helmert's gravity anomaly from the boundary to the surface. Downward continuation is then carried out by the inversion of this operator. It is shown that this can be done rigorously if, and only if, the processes of averaging and downward continuation are commutative (mutually interchangeable).

Le calcul d'un géoïde gravimétrique précis fondé sur l'approche Stokes-Helmert requiert la solution au problème géodésique de la valeur à la limite. Pour ce faire, l'anomalie gravimétrique moyenne d'Helmert sur la surface topographique de la Terre doit être réduite au géoïde, la surface qui joue le rôle de limite. Cette réduction est un processus connu comme étant la « réduction vers le bas ». Le présent article considère la réduction vers le bas comme une solution au problème de la loi de Poisson inverse discrète. L'article montre la dérivation de la double moyenne d'un opérateur de réduction vers le haut qui relie l'anomalie gravimétrique moyenne d'Helmert de la limite à la surface. La réduction vers le bas est ensuite réalisée par l'inversion de cet opérateur. Il est démontré que ceci peut être effectué rigoureusement si, et uniquement si, les processus de calcul de la moyenne et de réduction vers le bas sont commutatifs (mutuellement interchangeables).



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Introduction

A quantity $u(x,y,z)$ that satisfies the Laplace partial differential equation:

$$\nabla^2 (u) = 0 , \tag{1}$$

in a region of space is called *harmonic* in that region. Transforming the Cartesian coordinates x,y,z into the usual curvilinear coordinates $r,\varphi,\lambda \equiv r, \Omega$, we can define the direction "down" as going against the growth of r. In many geodetic applications, it is interesting to study the behaviour of a harmonic quantity (more generally the behaviour of a linear combination of a harmonic quantity and its vertical derivative) in the downward direction. For instance, the downward continuation of Helmert gravity anomalies from the Earth's topographic surface onto the boundary, the geoid, is a key process for the computation of a precise geoid following the Stokes-Helmert technique [*Vaníček and Martinec* 1994; Vaníček et al. 1999] formulated at and used by University of New Brunswick (UNB).

The downward continuation of a gravity anomaly to the geoid, a continuation process also known as the inverse Poisson problem, must precede the solution of the geodetic boundary value problem. By itself, it can be applied to several field quantities, for example, to observed gravity values (g), to gravity disturbances (δg), to disturbing potential (T), or any combination of these quantities. In the discussions that follow, the quantity of interest are Helmert's gravity anomalies measured on a mesh of cells at the Earth surface, and their corresponding mean cell values at the geoid. It is known [e.g., *Wong* 2002] that Helmert's gravity anomalies can be expressed as a linear combination of a harmonic quantity (T) and its vertical derivative $\left(\partial T / \partial r\right)$. Thus their downward continuation represents a proper inverse Poisson problem.

There are two different ways to compute downward continuation. One way is to formulate